Experimental Evaluation of the Effects of Manipulation by Merging in Weighted Voting Games

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Abstract: Weighted voting games are subject to a method of manipulation, called *merging*. This manipulation involves a coordinated action among some agents who come together to form a bloc by merging their weights in order to have more power over the outcomes of games. We conduct careful experimental investigations to evaluate the opportunities for beneficial merging available for strategic agents using two prominent power indices: *Shapley-Shubik* and *Banzhaf* indices. Previous work has shown that finding a beneficial merge is NP-hard for both the Shapley-Shubik and Banzhaf power indices, and leaves the impression that this is indeed so in practice. However, results from our experiments suggest that finding a beneficial merge is relatively easy in practice. Furthermore, while it appears impossible to stop manipulation by merging for a given game, controlling the quota ratio is desirable. Thus, we deduce that a high quota ratio reduces the percentage of beneficial merges. Finally, we conclude that the Banzhaf index may be more desirable to avoid manipulation by merging, especially for high quota ratios.

1 INTRODUCTION

Weighted Voting Games (WVGs) are classic cooperative games which provide compact representation for coalition formation models in human societies and multiagent systems. Each agent in a WVG has an associated weight. A subset of agents whose total weight is at least the value of a specified *quota* is called a *winning coalition*. The weights of agents in a game correspond to resources or skills available to agents, while the quota is the amount of resources or skills required for a task to be accomplished. For example, in *academia*, professors put their resources (i.e., weights) together to publish articles (i.e., quota). The relative power of each agent in WVGs reflects its significance in the elicitation of a winning coalition. To evaluate players' power in such games, prominent *power indices* such as the *Shapley-Shubik* (Shapley and Shubik, 1954) and *Banzhaf* (Banzhaf, 1965) indices are commonly used.

Recently, there is much interest in *manipulation* (i.e., dishonest behaviors) by strategic players in WVGs. These manipulations involve an agent or agents misrepresenting their identities in anticipation of gaining more power at the expense of other agents in a game. See (Bachrach and Elkind, 2008; Aziz and Paterson, 2009; Lasisi and Allan, 2010; Aziz et al., 2011; Lasisi and Allan, 2011). In manipulation by merging, which is also known as *alliance* or *collusion*, two or more agents voluntarily merge their voting weights to form a single bloc (Felsenthal and Machover, 1998; Felsenthal and Machover, 2002). In a beneficial merge, merged agents are compensated commensurate with their share of the power gained by the bloc. The agents whose weights are merged into a bloc are referred to as *assimilated* agents.

(Yokoo et al., 2005) consider collusion in open anonymous environments, such as the internet. They show that collusion in such environments can be difficult to detect. Thus, the increased use of online systems such as trading systems and peer-to-peer networks, where WVGs are also applicable, means that manipulation by merging remains an important challenge.

To provide insights into understanding the problem of manipulation by merging in WVGs, first, we recall that the problem of computing the Shapley-Shubik and Banzhaf indices is NP-hard (Matsui and Matsui, 2001). (Aziz et al., 2011) have also shown that determining if there exists a beneficial merge for the manipulators is NP-hard using either of the two indices to compute agents' power. Although this worst case complexity for manipulation by merging is daunting, it is possible that real instances of WVGs are easy to manipulate. We note that real WVGs are small enough that exponential amount of work may not deter manipulators from participating in manipulation by merging. Thus, according to (Keijzer et al., 2010), the number of players in most real life examples of WVGs is between 10 and 50. Hence, manipulations may, in some cases, be achieved in practice.

A careful investigation of effective heuristics for manipulating such games by merging are yet to be researched (Aziz and Paterson, 2009). This, we argue, may be primarily due to the inherent difficulty of the problem. This is because the ability to find beneficial merges depends on the characteristics of the game. Some games have little opportunity for merging while others could have many beneficial merges. So, in contrast to the work of (Aziz et al., 2011), in this paper, we study experimental evaluation of the effects of manipulation by merging using various parameters of the games to analyze opportunities for beneficial merging for the manipulators. This will provide insight into understanding the problem to both provide insights for heuristics and guide the decisions of game designers. Our evaluation is carried out using two prominent power indices: Shapley-Shubik and Banzhaf indices, to compute agents' power.

The NP-hardness results of finding a beneficial merge of the previous work for both the Shapley-Shubik and Bazhaf indices leave the impression that this is indeed so in practice. While finding the best merging may be difficult, results from our experiments suggest that finding a beneficial merge is relatively easy in practice. While we may be powerless to stop manipulation by merging for a given game, we suggest controlling *quota ratio*, which is the percentage of weight needed to form a winning coalition. The game designer may be able to control the quota ratio. Thus, we deduce that when the quota ratio of a game is high, the percentage of beneficial merges goes down. Finally, we conclude that the Banzhaf power index may be more desirable than the Shapley-Shubik power index to avoid manipulation by merging, especially for high values of the quota ratios.

2 RELATED WORK

(Felsenthal and Machover, 2002) characterize situations when it is advantageous or disadvantageous for agents to merge into a bloc, and show that using the Shapley-Shubik index, merging can be advantageous or disadvantageous. (Aziz and Paterson, 2009) focus on the complexity of finding advantageous merging. They show that for unanimity WVGs, it is disadvantageous for a coalition to merge using the Shapley-Shubik index to compute payoff. Also, determining if there exists a beneficial merge is NP-hard for the Shapley-Shubik index. (Lasisi and Allan, 2011) considers empirical evaluation of the extent of susceptibility of three indices, namely, Shapley-Shubik, Banzhaf, and Deegan-Packel indices to manipulation when agents engage in merging. Their results show that the Shapley-Shubik index is the most susceptible to manipulation via merging among the three indices. Furthermore, a recent work of (Lasisi and Allan, 2012) proposes a search-based approach to manipulation by merging. They show that manipulators need to do only a polynomial amount of work to find improved benefits when the size of the manipulators' bloc is restricted to a constant $2 \leq k < n$, where n is the number of agents in the WVG. The authors then present a pseudopolynomial-time enumeration algorithm that manipulators may use to find a much improved power gain over a random approach using both the Shapley-Shubik and Banzhaf indices to compute agents' power.

It is important to note that none of these papers deal with the experimental evaluation and analysis of the type of beneficial merging that we study here.

3 PRELIMINARIES

3.1 Weighted Voting Games

Let $I = \{1, \cdots, n\}$ be a set of n agents and the corresponding positive weights of the agents be $\{w_1, \cdots, w_n\}$. Let a coalition $S \subseteq I$ be a non-empty subset of agents. A WVG G, with quota q involving agents I, is represented as $G = [w_1, \ldots, w_n; q]$. We assume that $w_1 \geq w_2 \geq \ldots \geq w_n$. Note that this assumption does not affect the definition of the game or the generality of our results. Denote by $w(S)$, the weight of a coalition, S, derived as the summation of the weights of agents in S, i.e., $w(S) = \sum_{j \in S} w_j$. A coalition, S, wins in game G if $w(S) \geq q$, otherwise it loses. WVGs belong to the class of *simple voting games*. In simple voting games, each coalition, S, has an associated function $v : S \to \{0, 1\}$. The value 1 implies a win for S and 0 implies a loss. So, $v(S) = 1$ if $w(S) \geq q$, and 0 otherwise.

3.2 Power Vectors

(de Nijs and Wilmer, 2012) propose the use of *power vectors* to evaluate heuristics for the well-known inverse problem (Fatima et al., 2008; Keijzer et al., 2010) using the Banzhaf power index. We use power vectors in this paper to illustrate the effects of small changes in the weights of agents on their corresponding powers in WVGs.

Define a power vector for a WVG G of n agents as follows. Consider the weights of agents in G in a non-increasing order. The power vector of G is an n-dimensional vector $v \in \mathbb{R}^n$ of the power of each of the agents listed in order.

3.3 Shapley-Shubik Power Index

The Shapley-Shubik index quantifies the marginal contribution of an agent to the *grand coalition* (i.e., a coalition of all the agents). Each permutation of the agents is considered. We term an agent *pivotal* in a permutation if the agents preceding it do not form a winning coalition, but by including this agent, a winning coalition is formed. Shapley-Shubik index assigns power to each agent based on the proportion of times it is pivotal in all permutations. We specify the computation of the index using notation of (Aziz et al., 2011). Denote by π, a permutation of the agents, so $\pi : \{1,\ldots,n\} \to \{1,\ldots,n\}$, and by Π the set of all possible permutations. Denote by $S_\pi(i)$ the predecessors of agent i in π, i.e., $S_\pi(i) = \{j : \pi(j) < \pi(i)\}$. The Shapley-Shubik index, $\varphi_i(G)$, for each agent i in a WVG G is

$$\varphi_i(G) = \frac{1}{n!} \sum_{\pi \in \Pi} [v(S_\pi(i) \cup \{i\}) - v(S_\pi(i))]. \quad (1)$$

3.4 Banzhaf Power Index

An agent $i \in S \subseteq I$ is referred to as being *critical* in a winning coalition, S, if $w(S) \geq q$ and $w(S \backslash \{i\}) < q$. The Banzhaf power index computation for an agent i is the proportion of times i is critical compared to the total number of times any agent in the game is critical. The Banzhaf index, $\beta_i(G)$, for each agent i in a WVG G is given by

$$\beta_i(G) = \frac{\eta_i(G)}{\sum_{j \in I} \eta_j(G)} \quad (2)$$

where $\eta_i(G)$ is the number of coalitions for which agent i is critical in G.

4 PROBLEM DEFINITION

4.1 Problem Formalization

Let k and n be integers such that $2 \leq k \leq n$. Let $I = \{1, \cdots, n\}$ be a set of n agents and the corresponding weights of the agents be $\{w_1, \cdots, w_n\}$, where $w_i \in \mathbb{Z}$. Let $G = [w_1, \ldots, w_n; q]$ be a WVG of n agents with quota $q \in \mathbb{Z}$. Consider a manipulators' coalition C of k agents which is a k-subset of the n-set I. We assume that C contains distinct k elements chosen from I. Suppose the manipulators in C merge into a single bloc denoted by $\&C$, i.e., agents $i \in C$ have been assimilated into the bloc $\&C$, then, we have a new set of agents in the game after merging. Thus, the initial game G of n agents has been altered by the manipulators to give a new WVG G' of $n-k+1$ agents consisting of the bloc and other agents not in the bloc i.e., $I \backslash C$. Note that the weights of the non manipulators and the quotas in the two games remain the same.

Let ϕ be any of Shapley-Shubik or Banzhaf power index. Denote by $(\phi_1(G), \ldots, \phi_n(G)) \in [0,1]^n$ the power of agents in WVG G. Thus, for the manipulating agents $i \in C$ with power $\phi_i(G)$ in game G, the sum of the power of the k manipulators is $\sum_{i \in C} \phi_i(G)$, while that of the bloc formed by the manipulators in game G' is $\phi_{\&C}(G')$. The ratio $\tau = \frac{\phi_{\&C}(G')}{\sum_{i \in C} \phi_i(G)}$ gives a factor of the power gained or lost by the manipulators when they alter game G to give G'. The power index, ϕ, is said to be susceptible to manipulation in WVG G if there exists a G' such that $\tau > 1$; the merging is termed *advantageous* or *beneficial*. If $\tau < 1$, then the merging is *disadvantageous* or *non-beneficial*, while the merging is *neutral* when $\tau = 1$.

4.2 Examples of Merging in WVGs

We have used the Shapley-Shubik power index for illustration in these examples. The manipulators and their powers are shown in bold.

Example 1. *Beneficial Merging*

Let $G = [8, 8, \mathbf{8}, 6, 5, \mathbf{5}, \mathbf{4}, \mathbf{2}, \mathbf{2}, \mathbf{2}; 28]$ be a WVG of ten agents. The power vector of this game is $[0.167, 0.167, \mathbf{0.167}, 0.119, 0.099, \mathbf{0.099}, \mathbf{0.067}, \mathbf{0.039}, \mathbf{0.039}, \mathbf{0.039}]$. Thus, the manipulators' coalition $C = \{3, 6, 7, 8, 9, 10\}$. The cumulative power of these manipulators is 0.4481. Suppose the manipulators merge their weights to form a bloc $\&C$ and alter G to give $G' = [\mathbf{23}, 8, 8, 6, 5; 28]$. The power of the bloc is $\varphi_{\&C}(G') = \varphi_1(G') = 0.8000 > 0.4481$. The factor of power gained by the manipulators is $\tau = \frac{0.8000}{0.4481} = 1.8$.

Note that we have implicitly assumed that agents in the blocs formed are working cooperatively and have transferable utility. Thus, proceeds from merging can easily be distributed among the manipulators. For instance, in this example, each manipulators may first be assign a payoff equal to what it would get in the original game G, then, the gain (i.e., $0.8000 - 0.4481 = 0.3519$) derived from the altered game G' can then be distributed among the members of C using different solution concepts for revenue distribution from coalitional game theory.

Not all manipulation by merging are beneficial. Example 2 illustrates an example of a non-beneficial merge for the manipulators.

Example 2. *Non-beneficial Merging*

Let $G = [10, 9, 9, \mathbf{9}, 8, \mathbf{7}, 6, \mathbf{6}, \mathbf{2}, \mathbf{1}; 56]$ be a WVG of ten agents. The power vector of this game is $[0.135, 0.121, 0.121, \mathbf{0.121}, 0.118, \mathbf{0.118}, 0.118, \mathbf{0.118}, \mathbf{0.022}, \mathbf{0.008}]$. Thus, the manipulators' coalition $C = \{4, 6, 8, 9, 10\}$. The cumulative power of these manipulators is 0.3869. Suppose these manipulators merge their weights to form a single bloc $\&C$ and alter G to give $G' = [\mathbf{25}, 10, 9, 9, 8, 6; 56]$. The power of the bloc is $\varphi_{\&C}(G') = \varphi_1(G') = 0.3333 < 0.3869$. The factor of power lost by the manipulators is $\tau = \frac{0.3333}{0.3869} = 0.86$.

5 MERGING PREDICTION

5.1 Using Power Vectors

Using power vectors, we provide further examples to illustrate manipulation by merging. We have used power vectors to illustrate the effects of small changes in the weights of agents on their corresponding powers in WVGs. The Shapley-Shubik index is used in this example. The small changes in the weight of agents are related to weights changes when two or more agents merge their weights to form a bloc, thus providing some insights into merging. A crucial observation is that we can have many games having the same power vector. For example, the following WVGs: $[11, 9, 4; 12], [11, 8, 5; 12], [11, 7, 6; 12]$, $[10, 9, 5; 12]$, $[10, 8, 6; 12]$, and $[10, 7, 7; 12]$ all have the same power vector $[0.33, 0.33, 0.33]$, even though the weights' distribution of agents in the games differ.

In Figure 1, we consider all WVGs of 3 agents such that the total weights of the agents in each game is 24. Figures $1(a), 1(b)$, and $1(c)$ are for the cases when the quotas q of the games are 12, 16, and 18, respectively. The y-axis indicates the possible weights of the first agent while the x-axis indicates the possible weights of the second agents in the games. Note that since agents weights are given in non-increasing order, the possible weights for the second agent are dependent on the weights of the first agent. The possible weights of the third agents are not shown since they can implicitly be derived having known the weights of the first two agents.

Similar to only the 4 power vectors that are attainable in WVGs of 3 players using the Banzhaf index (de Nijs and Wilmer, 2012), there are also 4 different power vectors for these games when the number of agents is 3 and using the Shapley-Shubik index. These power vectors are coded as $1, 2, 3$, and 4 below:

1 : [0.33, 0.33, 0.33]

2 : [0.50, 0.50, 0.00]

3 : [0.67, 0.17, 0.17]

4 : [1.00, 0.00, 0.00]

with the games of each power vector representing appropriate regions shaded in Figure 1. It is easy to observe the following facts which have impacts on weight changes as it relates to merging:

- The number of different power vectors is a function of the number of agents, n, in the games.
- The size of the region (associated with a particular power vector) changes with the quota.
- Some weight vectors are volatile to changes with respect to small changes in weight (such as $[11, 7, 6; 18]$) while others are not (such as $[12, 11, 1; 18]$).

5.2 Difficulty of Merge Prediction

A visual description clarifies manipulation by merging in WVGs. We use the Shapley-Shubik power index for illustration. Consider a WVG of three agents denoted by the following patterns: Agent 1 (▥), Agent 2 (□), and Agent 3 (▤). The weight of each agent in the game is indicated by the associated length of the pattern. A box in the pattern corresponds to a unit weight. Each row represents a permutation. Suppose all permutations of the three agents are given as shown in Figure 2. We can use the same figure to consider a range of quotas from 1 to 6 for the game. The Shapley-Shubik indices of the three agents are computed from the figure and shown in the associated table of the figure. These power indices for the agents in the game correspond to using various values of the quota for the same weights of the agents.

Consider a manipulation where Agent 1 and Agent 3 merge their weights to form a new agent, say, Agent X. In this case, Agent 1 and Agent 3 cease to exist since they have been assimilated by Agent X. Thus, we have only two agents (Agent X and Agent 2) in the altered WVG. Figure 3 shows the results of the merging between Agent 1 and Agent 3. Notice that the number of rows has been reduced to two, as there are now only two possible orderings. Consider the cases when the quota of the game is 1 or 6, the power of the assimilated agents for Agent X from Figure 2 shows that Agent 1 and Agent 3 each has a power of $\frac{1}{3}$ for a total power of $\frac{2}{3}$. The power of

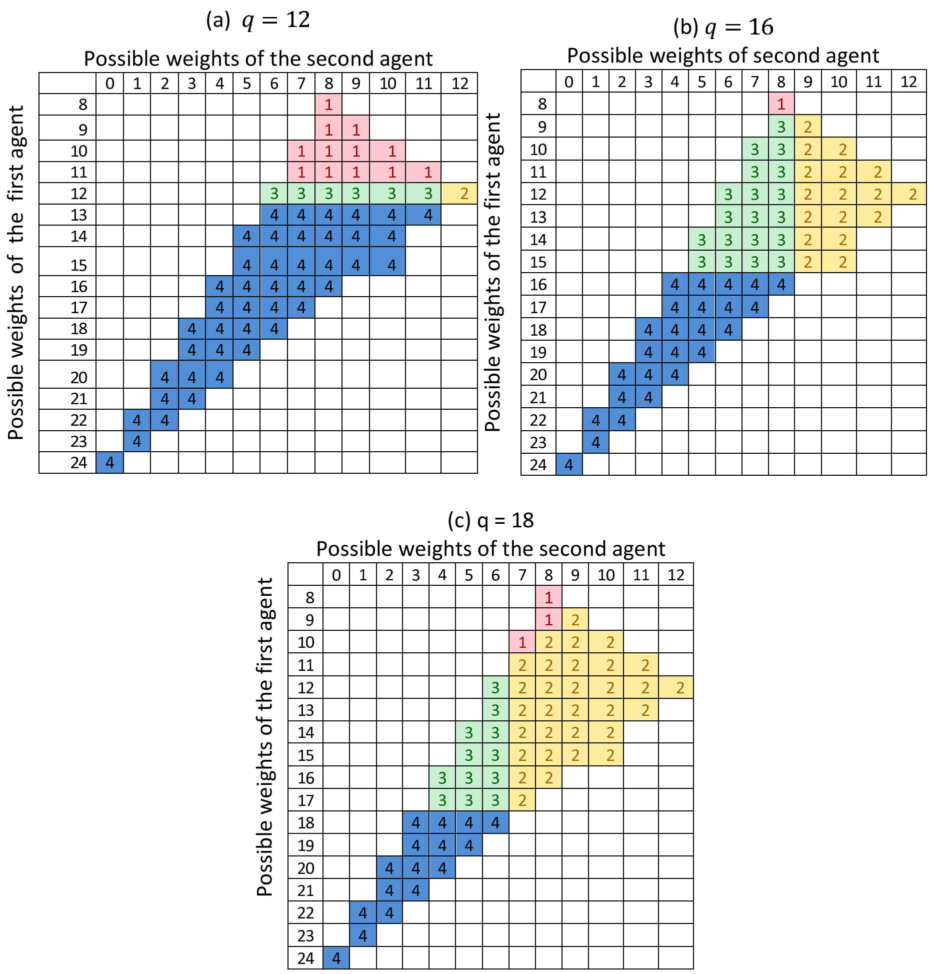

Figure 1: Using power vectors to illustrate the effects of small changes in the weights of agents on their corresponding Shapley-Shubik powers in WVGs.

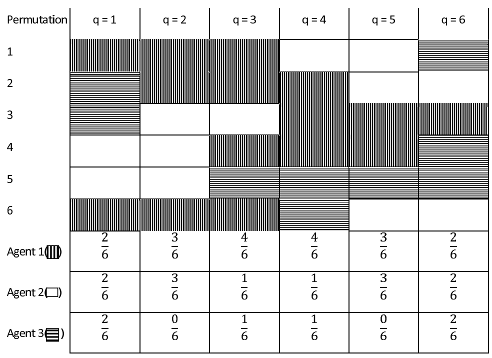

Figure 2: Six permutations of 3 agents and the power indices of the agents for values of quota from $q = 1$ to $q = 6$.

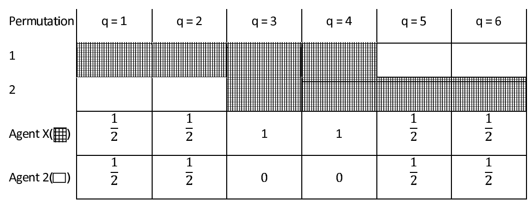

Figure 3: Manipulation by merging between Agent 1 and Agent 3 (from Figure 2) to form a new Agent X. The indices of Agent X and Agent 2 computed by Shapley-Shubik index after merging for various values of quota are also shown.

Agent X which assimilates these two agents in the two cases is each $\frac{1}{2} < \frac{2}{3}$. Also, the power of the manipulators stays the same for the cases where the quota is either 2 or 5. Specifically, the sum of the powers of Agent 1 and Agent 3 is $\frac{1}{2}$ for these cases. This is also true of Agent X for these cases. Finally, for the cases where the quota of the game is 3 or 4, the power of Agent X is 1 which is greater than $\frac{5}{6}$, the sum of the powers of Agent 1 and Agent 3 in the original game.

Note the difficulty of predicting what will happen when manipulators engage in merging. This illustration also shows that the choice of the quota of a game is crucial in determining the distribution of power of agents in a WVG. An apparent question that concerns the manipulators from the illustration above is the following : *Can effective merging heuristics be found even though predicting beneficial merging is difficult?*

6 EXPERIMENTS

6.1 Simulation Environments

We perform experiments to provide understanding and analysis of the opportunities for beneficial merging by manipulators in WVGs. We randomly generate WVGs. The weights of agents in each game are chosen such that all weights are integers and drawn from a Normal distribution, $N(\mu, \sigma^2)$, where μ and σ^2 are the mean and variance. We have used $\mu = 50$ and several values of standard deviation σ from the set $\{5, 10, \ldots, 40\}$. The number of agents, n, in each of the original WVGs is 10. For clarity of presentation, we have restricted the number of assimilated agents, k, in each game to 2. This is consistent with the assumptions of previous work on merging (Aziz et al., 2011; Lasisi and Allan, 2012) and coalition formation (Shehory and Kraus, 1998), as manipulators' blocs of small sizes are easier to form, and more importantly to the manipulators, they are less likely to be detected by other agents in the games. Apart from this, we also believe that an indepth understanding of this case (i.e., $k = 2$), will provide necessary background in understanding of the general case of when $k > 2$.

We have used a total of 200 distinct WVGs for our experiments. For each game, we vary the quota of the game from $\frac{1}{2}w(I) + 1$ to $w(I)$ in steps of 10, where $w(I)$ is the sum of the weights of all agents in the game. We then compute the factor of increment for each assimilated bloc of size 2 in a game using the two power indices. The evaluation is carried out for the proportion of beneficial merges in a game and the quota ratio, $\frac{q}{w(I)} = \frac{\text{quota}}{\text{total weight}}$. The quota ratios for the experiments range from 0.5 to 1.0, and indicate the fraction of the total weight needed for the quotas. A quota ratio of 1.0 suggests the existence of a type of WVGs referred to as *unanimity* WVGs, where all agents in a game are needed to form a winning coalition. Thus, a winning coalition always exists in the games. In other words, the quota ratio is a measure of the percentage of weight needed to form a winning coalition.

We consider all possible manipulators' blocs of size 2. The percentage of beneficial merge for a quota ratio is the fraction of cases whose factors of increment is greater than a specified value of τ. We tweak τ using different values to see how the percentage of beneficial merges varies and provide discussions of the effects noticed in the next subsection. The pseudocode to compute the percentage of beneficial merges for each quota ratio is given in Figure 4.

```
percentBeneficialMerge(Agents I, WVG G, τ) {
for quota q of G from ½w(I)+1 to w(I) step 10
    successCount = 0;
    totalCount = 0;
    for each manipulators' bloc b of size 2
      compute factor of increment f for b
       if f > τ then
         successCount++;
      totalCount++;
    end for
    quotaRatio = q / w(I);
   percentBenefit = successCount / totalCount;
 end for
}
```

Figure 4: Pseudocode to compute the percentage of beneficial merges for each quota ratio.

6.2 Discussion of Simulation Results

We present the results of our experiments. Figures 5 and 6 are indications of the opportunities for beneficial merging available for the manipulators. The x-axis is the quota ratio and the y-axis is the percentage of beneficial merging available to the manipulators when a beneficial merge is defined strictly as $\tau = 1$, i.e., a factor of power gain greater than 1.

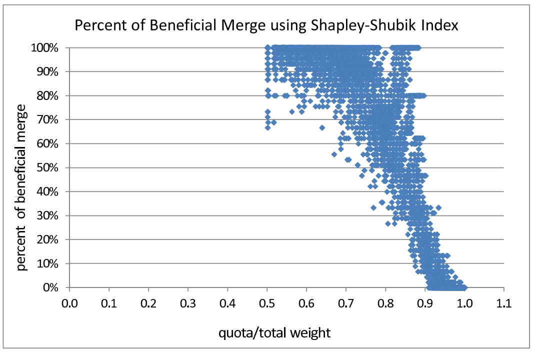

Figure 5: Percentage of beneficial merging for various values of quota ratio when a beneficial merge is defined to have a factor of power gain greater than 1.0 (Shapley-Shubik).

The theoretical results of (Aziz et al., 2011) on merging show that finding a beneficial merge is NP-hard for both the Shapley-Shubik and Bazhaf indices, and leave us with the impression that this is indeed so

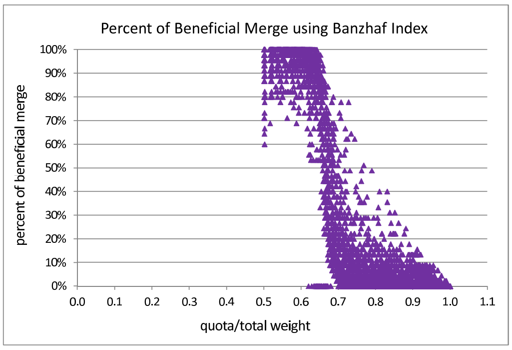

Figure 6: Percentage of beneficial merging for various values of quota ratio when a beneficial merge is defined to have a factor of power gain greater than 1.0 (Banzhaf index).

in practice. Figures 5 and 6 show that finding a beneficial merge is relatively easy in practice, at least for the WVGs we considered, and restricting each manipulators' blocs to size 2. In reality, finding the best merging may not even be desirable, as it assumes every agent will be willing to merge. Manipulators cannot petition every agent to see if they are willing to merge, as the manipulators would have announced their intent to cheat. However, a dishonest agent may first discover opportunities for beneficial merging before suggesting such merge to other would-be manipulators.

While it appears from the figures that we may be powerless to stop merging for a given game, the game designer may be able to control the quota. Thus, a high quota ratio reduces the opportunities for dishonesty as the percentage of beneficial merges goes down. Using the two indices to compute agents' power, we can deduce from Figures 5 and 6 that the Banzhaf index is more desirable to avoid cheating especially for high ratios. This is because the percentages of beneficial mergings for high values of the quota ratio using the Banzhaf index are smaller compare to those of the Shapley-Shubik index. Table 1 shows the means and standard deviations of the factor of power gained by manipulators from Figures 5 and 6. This shows that, on average, manipulation by merging is easier using the Shapley-Shubik index than using the Banzhaf index. This also indicates that the Banzhaf index may be more desirable to avoid manipulation in this situation.

For the second set of experiments, we consider a more realistic scenario for the manipulators. Even though we have defined a beneficial merge as a merge in which manipulators have a power gain with $\tau > 1$, manipulators may only be interested in beneficial merge with appreciable gains as the risks of being detected by the mechanism may exceed the anticipated benefits. We have restricted the minimal beneficial

Table 1: The means and standard deviations of the factor of power gained for Figures 5 and 6 using the two indices.

Indices	Shapley-Shubik	Banzhaf
Mean	1.142	1.062
Standard deviation	0.182	0.057

rate to $\tau = 1.05$ or 1.10, and do not notice appreciable change in the percentage of beneficial merging compared with those of Figures 5 and 6. Thus, we do not report them here.

However, for value of $\tau = 1.15$, which represents at least a 15% anticipated increment from the original power of the manipulators, we noticed a sharp contrast from earlier results. This is an interesting and a positive result for the designer of a game as it shows that the percentage of beneficial merges drops for both power indices. See Figures 7 and 8.

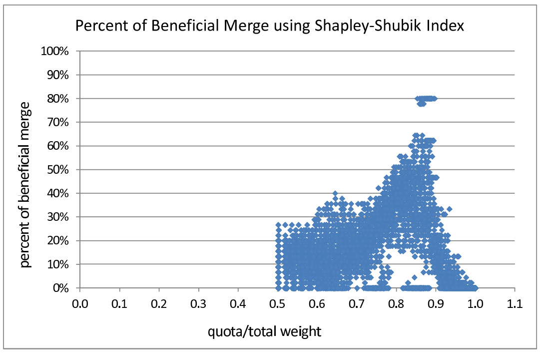

Figure 7: Percentage of beneficial merging for various values of quota ratio when a beneficial merge is defined to have a factor of power gain greater than 1.15 (Shapley-Shubik).

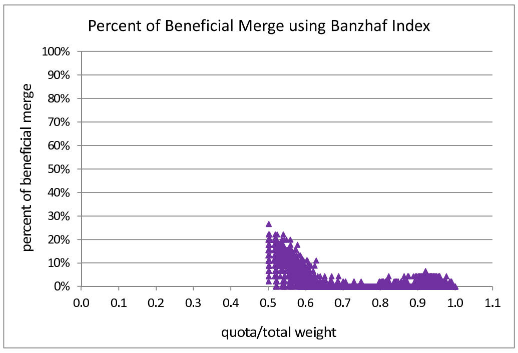

Figure 8: Percentage of beneficial merging for various values of quota ratio when a beneficial merge is defined to have a factor of power gain greater than 1.15 (Banzhaf index).

Consider Figure 7. The opportunities for beneficial merge for the manipulators using the Shapley-Shubik index may still be high, even when the factor of power gained has been increased to $\tau = 1.15$. However, for the case of the Banzhaf index (see Figure 8), the maximum percentage of beneficial merge available for the manipulators is considerably less. We argue that it is not unlikely that low percentage of beneficial merge may discourage manipulators in engaging in manipulation by merging if these conditions that we describe prevail and also using the Banzhaf power index to compute agents' power.

7 CONCLUSIONS

This paper considers experimental evaluation of the effects of manipulation by merging in weighted voting games. We conduct careful experimental investigations and analyses of the opportunities for beneficial merging available for strategic agents to engage in such manipulation using two well-known power indices, the Shapley-Shubik and Banzhaf power indices to compute agents' power. The following gives account of our main contributions.

First, we examine effects of small changes in the weights of agents on their corresponding powers in weighted voting games. This is illustrated by showing that power vectors are often unchanged. Second, we argue and provide empirical evidence to show that despite finding the optimal beneficial merge is an NP-hard problem for both the Shapley-Shubik and Banzhaf power indices, finding beneficial merge is relatively easy in practice. Hence, there may be little deterrent to manipulation by merging in practice using the NP-hardness results. Third, while it appears that we may be powerless to stop manipulation by merging for a given game, we suggest a measure, termed *quota ratio*, that the game designer may be able to control. Thus, we deduce that a high quota ratio decreases the number of beneficial merges. Finally, using the two power indices to compute agents' power, we conclude that the Banzhaf index may be more desirable to avoid manipulation by merging, especially for high values of quota ratios.

There are several areas of ongoing research on this problem. First, we seek to expand our experimental evaluations to consider and understand the general case of manipulators' blocs of size greater than 2. Second, we seek to understand the effects of other parameters of our experiments on the opportunities for beneficial merging for the manipulators. Finally, we also seek to provide careful investigations of effective heuristics for manipulation by merging.

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REFERENCES

Aziz, H., Bachrach, Y., Elkind, E., and Paterson, M. (2011). False-name manipulations in weighted voting games. *Journal of Artificial Intelligence Research*, 40:57–93.

Aziz, H. and Paterson, M. (2009). False-name manipulations in weighted voting games: splitting, merging and annexation. In *8th Intl. Conf. of AAMAS*, pages 409–416, Budapest, Hungary.

Bachrach, Y. and Elkind, E. (2008). Divide and conquer: false-name manipulations in weighted voting games. In *7th Intl. Conf. of AAMAS*, pages 975–982, Estoril, Portugal.

Banzhaf, J. (1965). Weighted voting doesn't work: a mathematical analysis. *Rutgers Law Review*, 19:317–343.

de Nijs, F. and Wilmer, D. (2012). Evaluation and improvement of Laruelle-Widgrén inverse Banzhaf approximation. *CoRR*, abs/1206.1145.

Fatima, S., Wooldridge, M., and Jennings, N. R. (2008). An anytime approximation method for the inverse Shapley value problem. In *7th Intl. Conf. of AAMAS*, pages 935–942, Portugal.

Felsenthal, D. S. and Machover, M. (1998). *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes*. Edward Elgar, Cheltenham.

Felsenthal, D. S. and Machover, M. (2002). Annexation and alliances: when are blocs advantageous a priori. *Social Choice and Welfare*, 19(2):295–312.

Keijzer, B., Klos, T., and Zhang, Y. (2010). Enumeration and exact design of weighted voting games. In *9th Intl. Conf. of AAMAS*, pages 391–398, Toronto, Canada.

Lasisi, R. O. and Allan, V. H. (2010). False name manipulations in weighted voting games: Susceptibility of power indices. In *13th Workshop on Trust in Agent Societies of AAMAS*, pages 139–150, Toronto, Canada.

Lasisi, R. O. and Allan, V. H. (2011). Annexations and merging in weighted voting games - the extent of susceptibility of power indices. In *3rd Intl. Conf. of ICAART*, pages 124–133, Rome, Italy.

Lasisi, R. O. and Allan, V. H. (2012). A search-based approach to annexation and merging in weighted voting games. In *4th Intl. Conf. of ICAART*, pages 44–53, Vilamoura, Algarve, Portugal.

Matsui, Y. and Matsui, T. (2001). NP-completeness for calculating power indices of weighted majority games. *Theoretical Computer Science*, 263((1-2)):305–310.

Shapley, L. S. and Shubik, M. (1954). A method for evaluating the distribution of power in a committee system. *American Political Science Review*, 48:787–792.

Shehory, O. and Kraus, S. (1998). Methods for task allocation via agent coalition formation. *Artificial Intelligence*, 101(1-2):165–200.

Yokoo, M., Conitzer, V., Sandholm, T., Ohta, N., and Iwasaki, A. (2005). Coalitional games in open anonymous environments. In *American Association for Artificial Intelligence*, pages 509–514.